Analysis the trapping electron by left circular polarization electromagnetic wave in static electric field

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Abstract

**Introduction: Electron Interaction with Electromagnetic Waves in Electric and Magnetic Fields**

The interaction of electrons with electromagnetic waves in the presence of external static electric and magnetic fields represents a fundamental problem in plasma physics and electromagnetic theory with profound implications for both basic science and technological applications [1,2]. This complex multi-field environment gives rise to rich nonlinear dynamics that govern phenomena ranging from particle acceleration in astrophysical plasmas to the operation of microwave devices and free-electron lasers [3].

When an electron moves through a uniform background magnetic field **B₀**, its trajectory becomes helical due to the Lorentz force, with the electron executing cyclotron motion at the characteristic cyclotron frequency ωc = eB₀/m, where e is the elementary charge and m is the electron mass [1]. The superposition of a static electric field **E₀** modifies this motion, introducing drift velocities and energy changes that can significantly alter the electron's response to incident electromagnetic radiation. This configuration creates a parameter space where resonant interactions can occur, leading to efficient energy transfer between the electromagnetic wave and the charged particle [4,5].

The theoretical framework for understanding these interactions builds upon the relativistic equations of motion for a charged particle in combined electromagnetic fields [5,6]. The presence of the static magnetic field introduces anisotropy into the system, with the electron's response to the electromagnetic wave becoming strongly dependent on the wave's polarization and propagation direction relative to the magnetic field vector. Meanwhile, the static electric field can provide secular acceleration or deceleration, fundamentally altering the electron's energy and thus its resonance conditions with the electromagnetic wave.

Several distinct regimes emerge depending on the relative magnitudes of the static fields and the wave amplitude. In the linear regime, where the wave fields are small compared to the static fields, the electron motion can be treated perturbatively, leading to well-defined absorption and emission coefficients [9]. However, as the wave amplitude increases, nonlinear effects become dominant, resulting in phenomena such as autoresonance, where the electron can maintain resonance with the wave despite changing energy, and stochastic motion, where the particle dynamics become chaotic [6,7].

The cyclotron resonance condition, ω - k‖v‖ = nωc (where ω is the wave frequency, k‖ is the parallel wave vector component, v‖ is the parallel electron velocity, and n is an integer), plays a central role in determining the efficiency of wave-particle interactions. The presence of the static electric field can shift electrons into and out of resonance, creating complex dynamical behavior that has been extensively studied both theoretically and experimentally.

These interactions have found numerous practical applications in plasma heating schemes for magnetic confinement fusion, where electron cyclotron resonance heating (ECRH) relies on precisely controlled electromagnetic waves to selectively heat electrons at specific locations within the plasma. Similarly, gyrotrons and other cyclotron resonance masers exploit these same physical principles to generate high-power millimeter and submillimeter wave radiation. In astrophysical contexts, similar processes are believed to be responsible for coherent radio emissions from pulsars and planetary magnetospheres.

Recent advances in computational plasma physics have enabled detailed numerical simulations of these multi-scale interactions, revealing new phenomena such as phase-space islands, separatrix crossing, and the formation of coherent structures in velocity space. These studies have enhanced our understanding of how energy and momentum are exchanged between electromagnetic fields and charged particles in complex field configurations.

The mathematical treatment of electron motion in crossed static electric and magnetic fields with superimposed electromagnetic waves requires sophisticated analytical and numerical techniques. The problem exhibits multiple characteristic time scales, from the fast cyclotron period to the slower drift time scales and the potentially much longer time scales associated with secular energy changes. This multi-scale nature necessitates careful analysis using techniques such as averaging methods, Hamiltonian perturbation theory, and direct numerical integration of the relativistic equations of motion.

**References**

1. Stix, T. H. (1992). *Waves in Plasmas*. American Institute of Physics Press, New York.
2. Ginzburg, V. L., & Syrovatskii, S. I. (1969). Cosmic ray origin and electron acceleration. *Annual Review of Astronomy and Astrophysics*, 7(1), 375-420.
3. Nusinovich, G. S. (2004). *Introduction to the Physics of Gyrotrons*. Johns Hopkins University Press, Baltimore.
4. Cairns, R. A. (1991). *Radiofrequency Heating of Plasmas*. Adam Hilger, Bristol.
5. Bekefi, G. (1966). *Radiation Processes in Plasmas*. John Wiley & Sons, New York.
6. Lichtenberg, A. J., & Lieberman, M. A. (1992). *Regular and Chaotic Dynamics*. Springer-Verlag, New York.
7. Karney, C. F. F. (1978). Stochastic ion heating by a lower hybrid wave. *Physics of Fluids*, 21(9), 1584-1599.
8. Fukuyama, A., Hayashi, K., & Takizuka, T. (1983). Fokker-Planck simulation of electron cyclotron heating in toroidal plasmas. *Nuclear Fusion*, 23(12), 1645-1653.
9. Bornatici, M., Cano, R., De Barbieri, O., & Engelmann, F. (1983). Electron cyclotron emission and absorption in fusion plasmas. *Nuclear Fusion*, 23(9), 1153-1257.
10. Albert, J. M. (2002). Nonlinear interaction of outer zone electrons with VLF waves. *Geophysical Research Letters*, 29(8), 21-1.